

CALCULUS II for BT: Midterm Exam - Some Old Questions

1. For $f(x, y) = \sin(x^2y)$, find the value of $f_{xy}(1, \frac{\pi}{2})$.

2. Given $xy + yz = xz^2$, find $\frac{\partial z}{\partial x}$.

3. Evaluate the double integral $\int_0^1 \int_0^2 ye^x dy dx$.

4. [10 marks] Evaluate the integral

$$\iint_R \frac{4x}{y^3 + 3} dA, \quad R = \{(x, y) : 0 \leq x \leq 2y, 1 \leq y \leq 2\}.$$

5. [20 marks] Find the critical points of the function $f(x, y) = 4xy - 2x^4 - y^2 + 4x - 2y$ and classify each as a local maximum, local minimum or saddle point.

6. The table below gives the values of a function $f(x, y)$ defined on $R = [0, 4] \times [0, 20]$.

(a) [10 marks] Estimate the value of $\iint_R f(x, y) dA$.

(b) [10 marks] Estimate the values of $f_x(3, 10)$ and $f_y(3, 10)$.

$x \backslash y$	0	5	10	15	20
0	1	1	2	3	2
1	1	4	6	8	7
2	2	7	12	14	12
3	3	11	17	21	19
4	4	15	23	27	24

7.(a) [10marks] Evaluate $\iint_D 3x^2 + 2y dx dy$, $D = \{(x, y) : 0 \leq x \leq 2, 0 \leq y \leq 1\}$.

(b) [10marks] Evaluate $\iint_D 6x + 2y dA$, D is bounded by the curves $y = 3x$, $y = x^2$.

(c) [10 marks] Find the volume of the solid under the surface $z = 5x^2 + 10y$ and above the region bounded by $y = x^2$ and $x = y^2$.

8. [15 marks] Using the given information, evaluate dz .

$$z = \ln(2x^2 + y^2), \quad x = 2, y = 1, dx = 0.01, dy = -0.02$$

SOLUTIONS

1. $-\pi$ 2. $\frac{y-z^2}{2xz-y}$ 3. $2(e-1)$ 4. $\frac{8}{3} \ln \frac{11}{4}$ 5. Saddle point at $(0, -1)$, local maxima at $(-1, -3)$ and $(1, 1)$. 6. (a) *Method 1* Using midpoints: 880; *Method 2* Using points furthest from the origin: 1135; *Method 3* Using points nearest to the origin: 565; *Best*: Take average of results of methods 2 and 3: 850. (b) $f_x(3, 10) \approx 5$ or 6 or average: 5.5; $f_y(3, 10) \approx 0.8$ or 1.2 or average: 1.0 7. (a) 10 (b) 729/10 (c) 27/14 8. 0.0044

10. For each pair of matrices below, find the product AB .

$$\text{(a) } A = \begin{bmatrix} 6 \\ 5 \\ 4 \end{bmatrix}, \quad B = \begin{bmatrix} -1 & 1 & 1 \end{bmatrix}, \quad \text{(b) } A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix},$$

11. [10 marks] For the matrices below, find $A + 2B$ and AB .

$$A = \begin{bmatrix} 1 & 3 & -2 \\ 0 & 2 & -3 \\ -2 & -1 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 0 & -1 \\ 2 & 3 & -2 \\ 3 & 1 & 2 \end{bmatrix}$$

12. [20 marks] Use the Gauss-Jordan method to solve the following system of equations:

$$\begin{aligned} x + 3y - 2z &= 1 \\ 3x - 2y + 4z &= 2 \\ 2x + 5y - 2z &= 3 \end{aligned}$$

13. [20 marks] Use the Gauss-Jordan method to solve the following problem:

100 toys are to be given out to a group of children. A ball costs \$2, a doll costs \$3 and a car costs \$4. A total of \$295 was spent. A ball weighs 12 oz, a doll 16 oz and a car 18 oz. The total weight of all the toys was 1542 oz. Find how many of each toy there are.

SOLUTIONS

12. $x = 0$, $y = z = 1$. **13.** 24 balls, 57 dolls, 19 cars.